Optimal Periodic Proof Test Based on Cost-Effective and Reliability Criteria

J. -N. Yang*
George Washington University, Washington, D.C.

An exploratory study for the optimization of periodic proof tests for fatigue-critical structures is presented. The optimal proof load level and the optimal number of periodic proof tests are determined by minimizing the total expected (statistical average) cost, while the constraint on the allowable level of structural reliability is satisfied. The total expected cost consists of the expected cost of proof tests, the expected cost of structures destroyed by proof tests, and the expected cost of structural failure in service. It is demonstrated by numerical examples that significant cost saving and reliability improvement for fatigue-critical structures can be achieved by the application of the optimal periodic proof test. The present study is relevant to the establishment of optimal maintenance procedures for structures in service.

I. Introduction

FATIGUE damage is one of the most important problems in the design of many structures, e.g., aircraft structures. For metallic structures, the fatigue damage is revealed by the initiation and propagation of cracks, which can be detected and repaired by scheduled inspection and maintenance procedures. As a result, the reliability of fatigue-critical structures depends not only on the structural design, but also on the maintenance procedures. The reliability analysis of structures with or without maintenance procedures recently has attracted increasing attention.[1-12]

In many practical applications, however, there are locations that are neither accessible for inspection nor inspectable, unless the structure is torn down where the tear-down maintenance procedure may be economically unfeasible. Therefore, repetitive proof testing at scheduled intervals becomes a promising alternative to prevent structural failure in service, and is, in fact, in current use.[13]

The significant advantage of proof testing in relation to structural reliability and optimum design has been discussed in the literature.[13-18] Recently, a fatigue reliability analysis of structures under random serive loads and periodic proof tests in service has been presented in Ref. 19. It is shown that significant reliability improvement for structures can be achieved by the application of periodic proof tests. It is further demonstrated that the reliability of structures increases as the number of periodic proof tests increases, or as the proof load level increases.

It should be noticed, however, that, as the number of proof tests increases, the cost of proof tests, including the total cost of performing proof tests, down time, nonavailability for service, etc., increases. Furthermore, as the proof load level increases, the probability of destroying structures under periodic proof tests increases; i.e., the average number of structures to be destroyed by periodic proof tests increases, thus increasing the cost of replacement. As a result, high level of structural reliability can be achieved through the application of periodic proof tests,[19] which in turn expenditure extra costs, i.e., the cost of proof tests and the cost of replacement. Consequently, there is a tradeoff potential between the structural reliability and the cost of proof tests.

It is the purpose of this paper to present an exploratory study for the optimization of periodic proof tests for fatigue-critical structures based on the cost-effective and reliability criteria. The optimal periodic proof test is obtained by minimizing an objective function, which is the total expected (statistical average) cost, while the constraint on the allowable level of structural reliability in service is satisfied. The total expected cost consists of the expected cost of proof tests (including the expected cost of performing proof tests, down time, nonavailability for service, etc.), the expected cost of structures destroyed by proof tests, and the expected cost of structural failure in service.

It is demonstrated by numerical examples that significant cost-saving and reliability improvement for structures can be achieved through the application of the optimal periodic proof test. The present effort is relevant to the establishment of rational design criteria, cost, and risk minimization, as well as the optimal maintenance procedure for fatigue-critical structures.

II. Formulation

The concept of the expected (statistical average) cost of failure in service has been used to obtain the optimal proof load level and the optimal design (thickness) of spacecraft pressure vessels, which are subjected to a single proof testing on the ground prior to the mission.[17] It also has been applied to determine the optimal inspection frequency for aircraft structures,[8] as well as the optimum design of other structures.[18] The concept of the expected cost of failure and the total expected cost will be used herein for the optimization of periodic proof tests.

The objective function to be minimized for the determination of the optimal proof load level and the optimal number of periodic proof tests is the total expected (statistical average) cost. The total expected cost EC^* considered herein consists of three parts; 1) the expected cost of performing the periodic proof tests, 2) the expected cost of structures to be destroyed by the proof tests, and 3) the expected cost of failure in service, including the loss of structure itself, mission degradation, loss of lives and equipment, etc.

$$EC^* = C_I\,[N+\bar{I}\,(r_0,N) + C_f P_f(r_0,N) + C_f P_f\,(r_0,N) \qquad (1)$$

in which

C_I = cost of performing one proof test for one structure

N = total number of scheduled periodic proof tests in the design service life

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*Associate Professor, Department of Civil, Mechanical, and Environmental Engineering.

$\bar{I}(r_0,N)$ = expected number of structures to be destroyed by the periodic proof tests in the service life; it is a function of the proof load level r_0 and the total number, N of periodic proof tests

C_2 = cost of one structure

C_f = cost of structural failure in service, including mission degradation, loss of structure itself, loss of lives and properties, etc.

$P_f(r_0,N)$ = probability of structural failure in the design service life, which is a function of the proof load level r_0 and the number N of scheduled periodic proof tests

The expected cost of performing proof tests $C_1[N+\bar{I}(r_0,N)]$ is proportional to the total expected number of proof tests, which consists of the total number N of scheduled proof tests, plus the expected number $\bar{I}(r_0,N)$ of structures destroyed by the proof tests. This is because, when a structure is destroyed by the proof test, a new structure is manufactured and proof-tested for replacement.

The second term in Eq. (1), $C_2\bar{I}(r_0,N)$ is the total expected cost of structures destroyed by the proof tests. The third term, $C_fP_f(r_0,N)$, is the expected cost of structural failure in service.

Dividing Eq. (1) by C_f, one obtains

$$EC=\gamma_1[N+\bar{I}(r_0,N)]+\gamma_2\bar{I}(r_0,N)+P_f(r_0,N) \tag{2}$$

in which

$$EC=EC^*/C_f \tag{3}$$

is the relative expected cost (or risk function) and

$$\gamma_1=C_1/C_f \qquad \gamma_2=C_2/C_f \tag{4}$$

In Eq. (4), γ_1 is the ratio of the cost of performing one proof test for one structure to the cost of structural failure in service; γ_2 is the ratio of the cost of a structure itself to the cost of structural failure in service. Both γ_1 and γ_2 are nondimensional quantities representing the relative importance, respectively, of both the cost of proof tests and the cost of replacement with respect to the cost of failure in service. They are important input parameters in the optimization process, and have to be estimated subjectively or objectively if possible.

The optimization problem considered herein is to find the optimal number N of periodic proof tests and the optimal proof load level r_0 so that the objective function EC, the relative expected cost (or risk function), is minimized, while the probability of failure of the structure in service should be less than a specified allowable level P_a. The optimization problem can be stated as follow; minimize the objective function EC, that is, the relative expected cost (risk function)

$$EC=\gamma_1[N+\bar{I}(r_0,N)]+\gamma_2\bar{I}(r_0,N)+P_f(r_0,N) \tag{5}$$

subject to the constraint

$$P_f(r_0,N)\leq P_a \tag{6}$$

where P_a is the allowable level of failure probability in service.

The mathematical techniques are available for solving the optimal solution r_0 and N for the system of equations given by Eqs. (5) and (6); for instance, the method of feasible direction[20] and nonlinear programming.[21] These classical techniques will not be discussed, and the interested reader is referred to, e.g., Refs. 20 and 21.

The estimation of the probability of failure $P_f(r_0,N)$ in service has been presented in Ref. 19. The expected number $\bar{I}(r_0,N)$ of structures to be destroyed by periodic proof tests will be derived in the next section.

III. Expected Number, $\bar{I}(r_0,N)$, of Structures Destroyed by Periodic Proof Tests

The ultimate strength R_0 of the new structure prior to service and proof testing is a random variable. Compilation of test results for aircraft structures[5] indicates that the distribution function $F_{R_0}(x)$ of R_0 can be approximated reasonably by a two-parameter Weibull distribution

$$F_{R_0}(x)=P[R_0\leq x]=1-\exp\{-(x/\beta_0)^{\alpha_0}\} \tag{7}$$

in which α_0 and β_0 are, respectively, the shape parameter and the scale parameter (or characteristic strength). It is found that[5] $\alpha_0=19$ is appropriate for aircraft structures.

The probability of failure B_0 of a new structure under the first (initial) proof test at a proof load level r_0 prior to service is, therefore,

$$B_0=F_{R_0}(r_0)=1-\exp\{-(r_0/\beta_0)^{\alpha_0}\} \tag{8}$$

Structures that pass the initial proof test are put into service. They are referred to as the original structures. The strength of the original structure remains unchanged in service until a fatigue crack is initiated, and then the residual strength decreases, owing to crack propagation, following Eq. (6) of Ref. 19. The time T_1 to fatigue crack initiation is a statistical variable with a Weibull distribution function $w(t)$ given by Eq. (5) of Ref. 19.

The specific type of service load considered herein is the flight-by-flight loading to aircraft structures, including gust loads, maneuver loads, ground loads, ground-air-ground loads, etc. Failure of the structure in service is assumed to occur when the ultimate strength or the residual strength is exceeded by service loads. Such a failure mode is referred to as the first-excursion or first-passage failure.[19,22-24]

Proof testing is performed at periodic intervals in service (see Fig. 1). Structures are eliminated or destroyed by the proof test when their residual strengths have degraded below the proof load level r_0. New structures are manufactured and proof-tested to replace those destroyed by the proof test, and they are referred to as the *renewal structures*. Hence, the strength of the structure after replacement is renewed, thus increasing structural reliability in service. Such a renewal process is taken into account in the present analysis.

The following formula for conditional probability is useful:

$$P[A]=\int_0^\infty P[A\mid T_1=t]w(t)\,dt \tag{9}$$

in which $P[A]$ is the probability of structural failure under the proof test, $P[A\mid T_1=t]$ is the conditional probability of failure under the proof test (under the condition that a fatigue crack is initiated at tth flight hour), and $w(t)dt$ is the probability that a fatigue crack is initiated in $(t,\ t+dt)$, given by Eq. (5) of Ref. 19.

Let B_j $(j=1,2,...,N-1)$ be the unconditional probability that an original structure will fail under the $j+1$th proof test performed at jT_0th flight hour. Then, B_j can be obtained from the conditional probability of structural failure under the $j+1$th proof test, $B_j(t)$ $(j=1,2,...,N)$, under the condition that the crack is initiated at t in $(0,T_0)$, by the application of Eq. (9). Note that $B_j(t)$ is given by Eqs. (12) and (13) of Ref. 19.

The probability B_1 that an original structure will fail under the second proof test performed at T_0 follows from Eq. (9) as

$$B_1=\int_0^{T_0} B_1(t)w(t)\,dt \tag{10}$$

in which $B_1(t)$ is the probability that an original structure will fail under the second proof test, under the condition that the fatigue crack is initiated at tth flight hour.

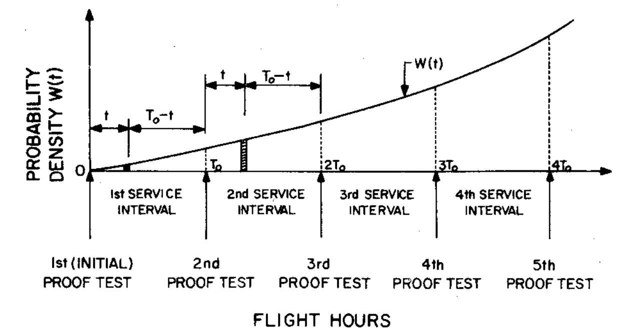

Fig. 1 Periodic proof tests, service intervals, and probability density of time to crack initiation.

For an original structure, the probability of failure B_2 under the third proof test consists of two parts

$$B_2 = \int_0^{T} B_2(t) w(t) \mathrm{d}t + \int_0^{T_0} B_1(t) w(T_0+t) \mathrm{d}t \qquad (11)$$

in which the first term is attributed to the event of crack initiation at t in the first service interval $(0,T_0)$. The second term is contributed by the event of crack initiation at $t+T_0$ in $(T_0,2T_0)$, in which case the probability of failure under the third proof test is equal to $B_1(t)$.

The probability of failure for an original structure under the fourth proof test is contributed by three events; 1) crack initiation at t in $(0,T_0)$, 2) crack initiation at T_0+t in $(T_0, 2T_0)$, and 3) crack initiation at $2T_0+t$ in $(2T_0, 3T_0)$, respectively, given in the following;

$$B_3 = \int_0^{T_0} B_3(t) w(t) \mathrm{d}t + \int_0^{T_0} B_2(t) w(T_0+t) \mathrm{d}t + \int_0^{T_0} B_1(t) w(2T_0+t) \mathrm{d}t \qquad (12)$$

In a similar manner, one obtains B_j as follows:

$$B_j = \sum_{i=1}^{j} \int_0^{T_0} B_{j-i+1}(t) w[(i-1)T_0+t] \mathrm{d}t \qquad (13)$$

Let L be the number of newly manufactured structures (prior to service) failed under the proof load r_0 before a structure that survives the proof load is obtained. Obviously, L is a statistical variable. The statistical distribution of L can be shown to follow a negative binomial distribution

$$P[L=\ell] = (1-B_0)B_0^{\ell} \qquad \ell=0,1,2,... \qquad (14)$$

in which B_0 is the probability of failure of the new structure under the proof load r_0 given by Eq. (8), and $(1-B_0)$ is the probability of surviving r_0.

The expected (average) number $\bar{L}$ of new structures to fail under the proof load r_0 before one is found which survives r_0 can be computed by use of Eq. (14):

$$\bar{L} = \sum_{\ell=0}^{\infty} \ell(1-B_0)B_0^{\ell} = B_0/(1-B_0) \qquad (15)$$

Let $\bar{I}_j$, $j=1,2,3...,N$ be the expected (average) number of structures destroyed by the jth proof test performed at $(j-1)T_0$th flight hour. Then, the expected number of structures destroyed under the initial (first) proof test prior to service $\bar{I}_1$ is obvious:

$$\bar{I}_1 = \bar{L} = B_0/(1-B_0) \qquad (16)$$

The expected number of structures to fail under the second proof test performed at T_0 is attributed to two different populations: the original structure and the new structure manufactured at T_0 when the original structure fails under the second proof test. Hence,

$$\bar{I}_2 = B_1 + B_1[B_0/(1-B_0)] = B_1/(1-B_0) \qquad (17)$$

in which B_1 is the probability that the original structure will fail under the second proof test performed at T_0, and is given by Eq. (10). Since we are concerned with one structure, $B_1 \times 1$ is the expected number of original structures to be destroyed by the second proof test; $B_0/(1-B_0)$ is the expected number of failure for the renewal structure manufactured at T_0 when the original structure fails (with probability B_1).

The expected number of structures to be destroyed by the third proof test performed at $2T_0$, comes from three different populations; 1) the original structure, which survives the second proof test, 2) the renewal structure manufactured at $2T_0$ when the original structure fails under the third proof test, and 3) the renewal structure manufactured at T_0. These three contributions are given, respectively, in the following:

$$\bar{I}_3 = B_2 + [B_2B_0/(1-B_0)] + B_1\bar{I}_2 = [B_2/(1-B_0)] + B_1\bar{I}_2 \qquad (18)$$

in which B_2 is given by Eq. (11), and $\bar{I}_2$ is given by Eq. (17).

The first term $B_2 \times 1$ is the expected failure attributed to the original structure, and the second term is the expected failures attributed to the renewal structure manufactured at $2T_0$ when the original structure fails at $2T_0$ (with probability B_2). The expected number of failures attributed to the renewal structures manufactured at T_0 is $\bar{I}_2$, and the probability of having such a renewal structure is B_2. Hence, $B_1\bar{I}_2$ is the contribution from the renewal structure manufactured at T_0.

The expected (average) number of structures $\bar{I}_4$ to be destroyed by the fourth proof test, performed at $3T_0$, is contributed by 4 different populations: 1) the original structure, which has survived all of the previous proof tests; 2) the renewal structure manufactured at $3T_0$ when the original structure fails under the fourth proof test at $3T_0$ (with probability B_3); 3) the renewal structure manufactured at T_0 for replacing the original structure (with probability B_1); and 4) the renewal structure manufactured at $2T_0$ for replacing the original structure (with probability B_2). Hence,

$$\bar{I}_4 = B_3 + B_3[B_0/(1-B_0)] + B_1\bar{I}_3 + B_2\bar{I}_2 = [B_3/(1-B_0)] + B_1\bar{I}_3 + B_2\bar{I}_2 \qquad (19)$$

in which B_3 is given by Eq. (12). In a similar manner, it can be shown that the expected number of structures to be destroyed by the jth proof test is

$$\bar{I}_j = \frac{B_{j-1}}{1-B_0} + \delta_{j-2} \sum_{k=1}^{j-2} B_k \bar{I}_{j-k} \qquad (20)$$

in which $\delta_{j-2}=0$ for $j \leq 2$ and $\delta_{j-2}=1$ for $j>2$.

The total expected number of structures to be destroyed by the periodic proof tests in the design service life $(0,NT_0)$ is, therefore,

$$\bar{I}(r_0,N) = \sum_{j=1}^{N} \bar{I}_j \qquad (21)$$

Substitution of Eq. (20) into Eq. (21) yields

$$\bar{I}(r_0,N) = \sum_{j=1}^{N} \frac{B_{j-1}}{1-B_0} + \sum_{j=3}^{N} \sum_{k=1}^{j-2} B_k \bar{I}_{j-k} \qquad (22)$$

where $\bar{I}_{j-k}$ is given by Eq. (20). Note that both $\boldsymbol{B}_0$ and $\boldsymbol{B}_j$ are functions of r_0, and hence, $\bar{I}(r_0,N)$, given by Eq. (22), is a function of r_0 and N.

The solution for $\bar{I}(r_0,N)$, derived in Eq. (22), holds for the renewal policy; i.e., a new structure is manufactured and proof-tested to replace the structure destroyed by the proof test. For the nonrenewal policy, the solution for $\bar{I}(r_0,N)$, given by Eq. (22), still holds, except that the second term (double sum) should be disregarded, since it represents the expected number of failures under proof tests due to the renewal structures.

It should be mentioned that the expected (average) number $\bar{I}(r_0,N)$ of structures to be destroyed by periodic proof tests derived in Eq. (22) is for one airplane (or one structure). When a fleet of m airplanes (structures) is considered, the expected number of structures in a fleet to be destroyed by periodic proof tests is $m\,\bar{I}(r_0,N)$. As a result, $100\,\bar{I}(r_0,N)$ represents the expected (average) percentage of the total structures (in a fleet) to be destroyed by periodic proof tests. Furthermore, under the renewal policy, $100\,\bar{I}(r_0,N)$ is also the expected percentage of replacement for a fleet of airplanes (structures).

IV. Numerical Examples

Example 1: Transport-Type Aircraft

The same numerical example for a critical component of the transport-type aircraft structure (presented in Ref. 19) is considered, in which the design service life T for the airplane is 15,000 flight hours. The probability of failure $P_f(r_0,N)$ within the design service life of 15,000 flight hours has been displayed in Fig. 3 of Ref. 19. It is observed that $P_f(r_0,N)$ decreases as the proof load level r_0, or the number N of periodic proof tests increases.

The expected percentage, $100\bar{I}(r_0,N)$ [Eq. (22)], of structures to be destroyed by periodic proof tests in the design service life of 15,000 flight hours is plotted in Fig. 2; this figure also represents the expected percentage of replacement under the renewal policy. It is observed that $\bar{I}(r_0,N)$ increases as the proof load level r_0 increases, or as the number N or periodic proof tests increases.

The relative expected cost EC [Eq. (5)], is plotted in Figs. 3a-3c for various values of γ_1 and γ_2. Also plotted in these

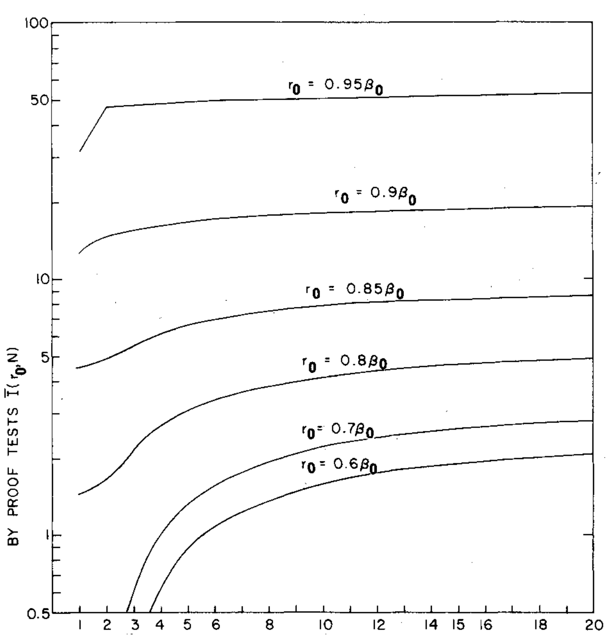

Fig. 2 Expected percentage of structures to be destroyed by periodic proof tests.

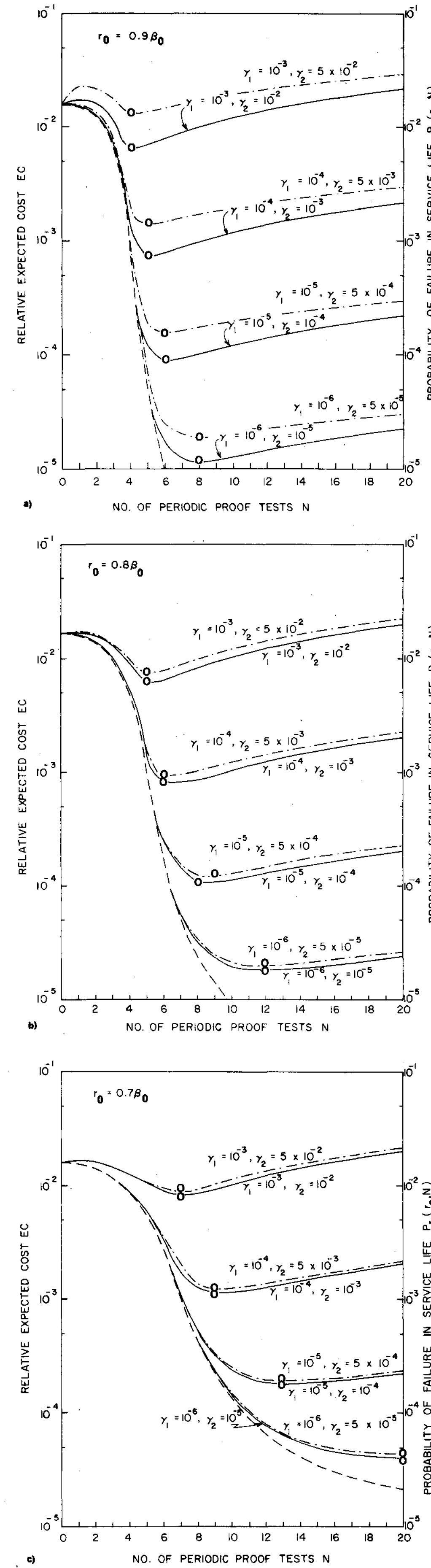

Fig. 3 Relative expected cost vs no. of periodic proof tests for various proof load levels; a) $r_0 = 0.9\beta_0$, b) $r_0 = 0.8\beta_0$, c) $r_0 = 0.7\beta_0$.

figures, as dashed curves, are the probability of structural failure $P_f(r_0,N)$ in the design service life. Several interesting observations are given in the following:

1) For a chosen proof load level r_0 and given values of γ_l and γ_2, there is an optimal number N for the periodic proof test at which the relative expected cost EC is minimum. It is observed from Fig. 3 that each curve has a minimum, as indicated by a circle.

2) The optimal number N of the periodic proof test for a chosen proof load level r_0 increases as the values of γ_l and γ_2 decrease, indicating that, as the costs of proof testing and replacement decrease, the optimal number of periodic proof tests increases. This is consistent with our intuition that the less expensive the proof testing and the cost of replacement are, the more one can afford to perform a greater number of proof tests, to achieve a higher level of structural reliability.

For a chosen proof load level r_0 and given values of γ_l and γ_2, the local minimum, as indicated by a cricle in Fig. 3, may or may not be the optimal solution, depending on the constraint of the allowable probability of failure P_a given by Eq. (6). Since the probability of failure $P_f(r_0,N)$ also is plotted in Fig. 3, it can be determined easily whether the local minimum is the optimal solution. For instance, if $P_a = 10^{-3}$ (reliability 0.999), it is clear from Figs. 3a-3c that, for $r_0 \geq 0.7\beta_0$, all of the minima are optimal solutions for various γ_l and γ_2 values. In such a situation, the constraint of Eq. (6) is said to be inactive, indicating that the inequality holds for Eq. (6).

Previous discussions are based on the premise that a proof load level is preselected. For some practical applications, it may be desirable to determine both the optimal proof load level r_0 and the optimal number N of periodic proof tests for given values of γ_l and γ_2. In order to accomplish this objective, the optimal solutions (circles) in Figs. 3a-3c are plotted in Fig. 4. The optimal solution then can be determined directly, as shown by the circles. For instance, for $P_a = 10^{-3}$, $\gamma_l = 10^{-3}$, $\gamma_2 = 10^{-2}$, the optimal solution is $r_0 = 0.85\beta_0$, $N = 5$.

The solid curves in Figs. 3 and 4 are for a set of γ_l and γ_2 values, with $\gamma_l/\gamma_2 = 0.1$. Under this situation it is observed that, as both γ_l and γ_2 decrease, the optimal proof load level r_0 increases from $0.85\beta_0$ to $0.9\beta_0$, whereas the optimal number N of periodic proof tests increase from 5 to 8 (Fig. 3). Hence, the optimal proof load level r_0 is between $0.85\beta_0$ and $0.9\beta_0$, whereas the optimal number of proof tests N is between 5 and 8.

A set of dashed-dot curves given in Fig. 3, and another set of dashed curves appearing in Fig. 4, are for the situation in which the ratio of γ_l/γ_2 is 0.02. It can be observed from Fig. 4 that the optimal proof load level, as indicated by circles, is, in general, smaller than that associated with the solid curves. This is consistent with the reasoning that the higher the cost of replacement, the less we can afford to destroy structures under proof tests; hence, the proof load level should decrease, since high proof load level tends to destroy more structures, as indicated by Fig. 2.

General conclusions derived from the observation of Figs. 3 and 4 are that 1) the optimal number N of periodic proof tests increases as both the cost of performing proof testing and the cost of replacement decrease and 2) the optimal proof load level r_0 decreases as the cost of replacement increases.

Another significant observation is that the optimal solution is not critical to the proof load level, in the sense that EC is a slowly varying function of r_0. Hence, if we do not choose exactly the optimal proof load level, the increase in the relative cost EC is not significant. For instance, the optimal solution for $\gamma_l = 10^{-3}$, $\gamma_2 = 10^{-2}$ is $r_0 = 0.85\beta_0$, $N = 5$, with $EC = 0.6 \times 10^{-2}$. If we choose $r_0 = 0.8\beta_0$, $N = 5$, then EC is 0.63×10^{-2}, whereas $EC = 0.64 \times 10^{-2}$ for $r_0 = 0.9\beta_0$, $N = 4$. This conclusion is of practical importance, and is very beneficial to the planning of proof tests, since we may choose a preferable level of proof load without much penalty. On the other hand, however, the relative cost EC is sensitive to the change of N, the number of the periodic proof tests, as indicated by each curve of Fig. 3. As a result, the designer should be careful in choosing the optimal number N for periodic proof tests. It is clearly observed from Fig. 3 and 4, as well as from Fig. 3 of Ref. 19, that significant cost saving and reliability improvement for structures have been achieved by the application of the optimal periodic proof test in service.

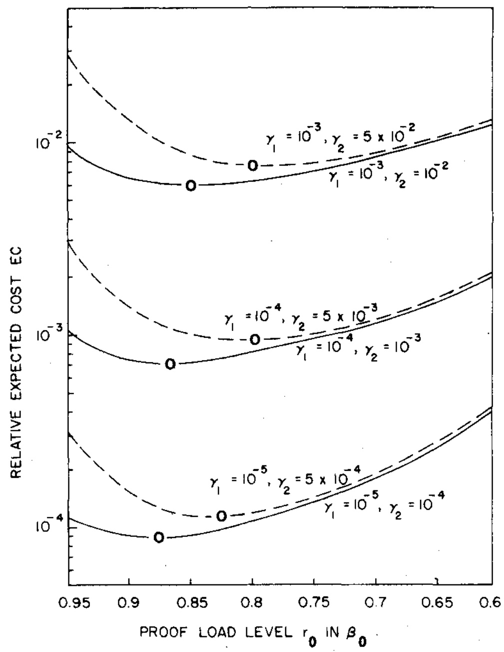

Fig. 4 Relative expected cost vs proof load level for optimal no. of proof tests.

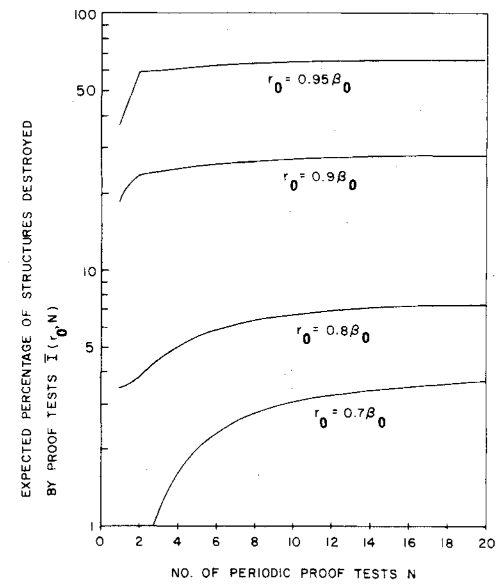

Fig. 5 Expected percentage of structures to be destroyed by periodic proof tests.

Example 2: Fighter Aircraft

The same numerical example for a critical component of a fighter aircraft wing subjected to F-111 maneuver loading spectra presented in Ref. 19 is considered. The probability of failure $P_f(r_0,N)$, within the design service life of 1500 flights, has been presented in Fig. 5 of Ref. 19. It is observed that $P_f(r_0,N)$ decreases as the proof load level r_0, or the number of proof tests N increases.

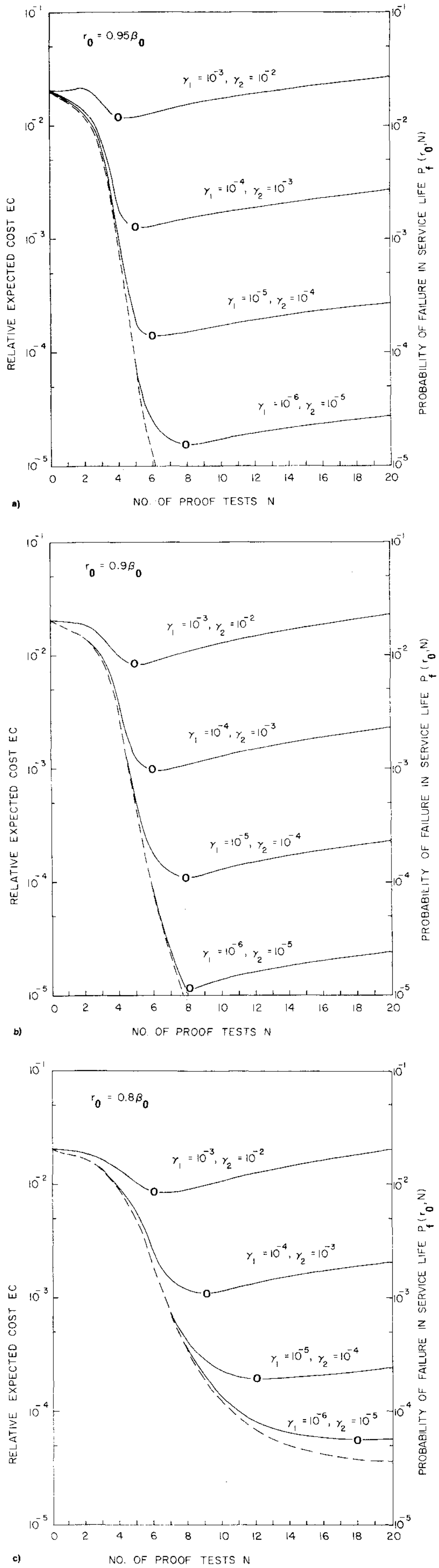

Fig. 6 Relative expected cost vs no. of periodic proof tests for various proof load levels; a) $r_0 = 0.95\beta_0$, b) $r_0 = 0.9\beta_0$, c) $r_0 = 0.8\beta_0$.

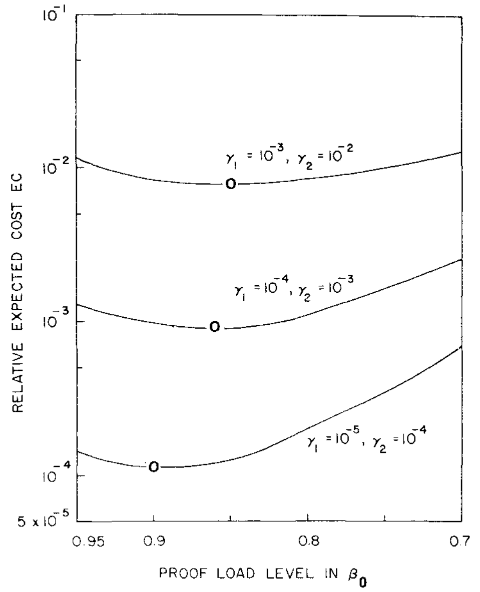

Fig. 7 Relative expected cost vs proof load level for optimal no. of proof tests.

The expected percentage, $100\bar{I}(r_0,N)$ [Eq. (22)], of structures to be destroyed by periodic proof tests in the design service life of 1500 flights is plotted in Fig. 5. Under the renewal policy, Fig. 5 also represents the expected percentage of replacement. As can be observed from Fig. 5, $\bar{I}(r_0,N)$ increases as the proof load level r_0 increases, or as the number N of periodic proof tests increases.

The relative expected cost EC [Eq. (5)], is plotted in Figs. 6a-6c for various values of γ_1 and γ_2. Also plotted as dashed curves in Fig. 6 are the probabilities of structural failure $P_f(r_0,N)$ in the design service life. The observations made in Example 1 for transport-type aircraft also prevail in the case of fighter aircraft as follows: 1) For a chosen proof load level r_0, and given values of γ_1 and γ_2, there is an optimal number N for the periodic proof tests at which the relative expected cost EC is minimum; and 2) for a chosen r_0, the optimal number N of periodic proof tests increases as the values of γ_1 and γ_2 decrease, indicating that when the cost of proof testing and the cost of replacement decrease, the optimal number of proof tests increases, thus increasing the structural reliability.

For some practical applications, instead of choosing a proof load level a priori, it may be desirable to determine both the optimal proof load level and the optimal number of periodic proof tests. Therefore, the optimal solutions (circles) appearing in Fig. 6 are plotted in Fig. 7. The optimal proof load level can then be determined directly from Fig. 7, as indicated by the circle.

The general trends regarding the optimal periodic proof test observed in Example 1 for transport-type aircraft also have been found herein: 1) As the cost of proof testing γ_1 and the cost of replacement γ_2 decrease, both the optimal proof load level r_0 and the optimal number N of the periodic proof test increase. 2) As the cost of replacement γ_2 increases, the optimal proof load level r_0 decreases.

It is observed from Fig. 7 that the optimal periodic proof test is not critical to the proof load level, in the sense that EC varies slowly with respect to r_0. As a result, if a preferable proof load level that is not optimal is chosen, the cost penalty is not severe as long as the optimal number of periodic proof tests is used. Such a conclusion is of practical significance and is very beneficial to the planning of periodic proof tests. Finally, it is observed from Figs. 6 and 7, as well as from Fig. 5 of Ref. 19, that the application of the optimal periodic proof test results in a significant cost saving and reliability improvement for structures.

V. Discussion and Conclusion

A formulation for the optimization of periodic proof tests for fatigue-critical structures has been presented. The optimal proof load level and the optimal number of periodic proof tests are obtained by minimizing the total expected cost, while the constraint on structural reliability in service is satisfied.

Numerical results indicate that the optimal number of periodic proof test increases as the cost of proof tests and the cost of replacement decrease, thus increasing structural reliability in service. It is shown that the optimal proof test is not critical to the proof load level, so that one may be able to choose a preferable proof load level based on other considerations without significant cost penalty. For instance, the proof load level should not exceed the yield stress if considerable yield strain may occur after yielding for a particular material. Therefore, the conclusion obtained herein is very beneficial to the planning of periodic proof tests. Furthermore, it has been demonstrated that significant cost savings and reliability improvement for fatigue-critical structures can be achieved by the application of the optimal periodic proof test.

For the sake of simplicity, we have restricted ourselves to periodic proof tests, i.e., constant interval tests with constant proof load levels r_0 throughout the service life. It has been observed in Ref. 19 and indicated in Ref. 8 that both the proof tests and the inspection and repair maintenance procedures at the later service life, i.e, near the expected fatigue life, are more efficient and beneficial in reliability improvement, and thus cost saving. Consequently, the proof test intervals and the proof load levels should vary in order to achieve maximum benefit in the optimization process, e.g., longer interval in the early service life and shorter interval in later serivce life. It should be noted that our restriction can be relaxed to include variable intervals and variable proof load levels without any theoretical difficulty. However, the number of variables to be optimized increases, thus increasing the complexity of numerical computation. This is an interesting subject for further investigation.

A very constructive idea for economic criteria, as suggested by one of the reviewers, is the use of interest rate to bring cost to a common time base in the optimization formulation. This is particularly relevant to civil engineering structures, such as offshore platforms. In fact, this idea has been applied recently in Ref. 26 for optimal aseismic design of buildings. It is an interesting subject for further study for aircraft structures.

Our major objective in the present study is the establishment of an optimal proof test maintenance procedure in the design stage prior to service. As a result, service loads to aircraft structures, such as gust loads, maneuver loads, etc., are based on past experience and specifications (see Ref. 25). Both the static and fatigue strength are estimated from results of laboratory coupon and full-scale tests, as well as analyses. A problem of practical importance is to establish a strategy for proof test based on known (measured) stress and load history in service, up to the point of proof test or inspection. This type of strategy is of particular importance when the actual aircraft usage deviates from what is expected in the design stage. In this connection, the Baysian approach should be employed to update the information, such as measured loads in service.[18] This subject, however, is beyond the scope of the present study.

In the reliability analysis[19] and present optimization study, many statistical variables have been accounted for, among which service loads to aircraft, and ultimate, residual, and fatigue strength of structures involve considerable statistical variability. The exceedance characteristics of service loads, as specified in Ref. 25, have been used[19]; they reflect the frequency distribution of service loads, and have been established through the statistical analysis of extensive measured service data. The Weibull distribution has been used for the ultimate strength, residual strength, and fatigue strength of aircraft structures.[19] The Weibull distribution has been established through the compilation of existing full-scale static data,[5] as well as through extensive laboratory fatigue data on coupons and full-scale structures (see Refs. 5, 7, 9, 11, etc., for detailed discussions). Likewise, from the physical standpoint, Weibull frequency distribution is a logical choice for the static and fatigue strength of structures, since it can be derived from the weakest link hypothesis or the wear-out model.

Nevertheless, there may be a concern on the sensitivity of the optimal strategy with respect to a particular choice of frequency distribution for each statistical variable involved. The answer to such a concern is that the optimal strategy is not sensitive at all to the particular choice of the distribution function for structural strength. Note that the proof test screens out structures having weak strength and truncates the distribution functions of both the ultimate strength and the residual strength at the lower tail up to the proof load level (see derivation in the Appendix of Ref. 19 and Fig. 1 of Ref. 17). This truncation eliminates precisely the portion of strength distribution which is least known and which has greatest interaction with service loads (i.e., greatest contribution to failure probability). Therefore, the proof test alleviates analytical difficulty in justifying the validity of a fitted frequency distribution in the lower tail portion, where experimental data usually are nonexistent, thus increasing the statistical confidence in the reliability analysis and optimization procedures. This is an important advantage of proof test, and it has been emphasized in Refs. 17-18.

Consequently, the sensitivity of the optimal strategy lies mainly on the accuracy of the selected (fitted or assumed) frequency distribution for service loads. This problem is relieved for gust loads to transport-type aircraft, since extensive measured data are available,[25] as discussed previously. For other types of structures, such as civil engineering structures under earthquakes or tornados, care should be exercised in choosing a particular distribution function. This, however, is a classical difficulty in the area of structural reliability, where extensive data do not exist.

Although emphasis is placed on aircraft structures in the present study, the principle involved can be applied equally to many other engineering disciplines. In particular, the optimal strategy can be highly significant for a large electronic system, which consists of a large number of components to be proof-tested. Since the cost of each electronic component is so small in comparison with the total cost of the entire large system that the optimal (high) proof load level and optimal (short) proof test interval will result in outstanding system reliability, as well as significant cost savings.

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